3.2 Electric Field Intensity

The electric field intensity $\bar{E}$ due to Q is defined as

"The force per unit charge on the test charge q"

$$\bar{E} = \frac{\bar{F}}{q} \quad \text{N/C } or \text{ V/m}$$

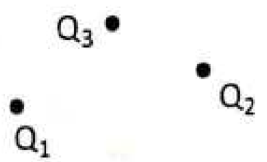
The electric field due to a point charge is

$$\bar{E}_Q = \frac{Q}{4\pi\epsilon R^2}\,\bar{a}_R \quad \text{V/m}$$

3.3 Charge distribution

(1) Point Charges

- Are assumed to exist at isolated points in space as shown in fig



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(2) Line charge distribution

- The charge is distributed over a line with a linear charge density ρ_L (C/m)
- The total charge on the line

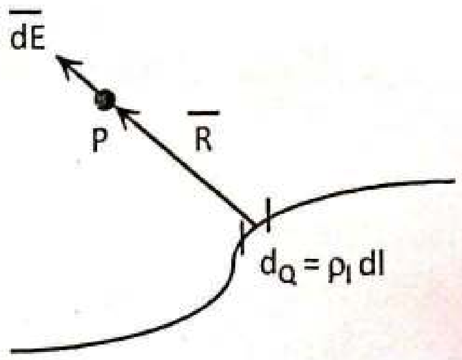
$$Q = \int_L \rho_l \, dl$$

- Each differential charge dQ along the line produces a differential electric field at P

$$d\bar{E} = \frac{dQ}{4\pi\epsilon R^2}\,\bar{a}_R = \frac{\rho_l \, dl}{4\pi\epsilon R^2}\,\bar{a}_R$$

- The total field at P is

$$\bar{E} = \int d\bar{E} = \int_L \frac{\rho_l \, dl}{4\pi\epsilon R^2}\,\bar{a}_R$$



(3) **Sheet charge distribution "Surface"**

- The charge is distributed over a surface with a surface charge density ρ_s (C/m^2)
- The total charge on the surface

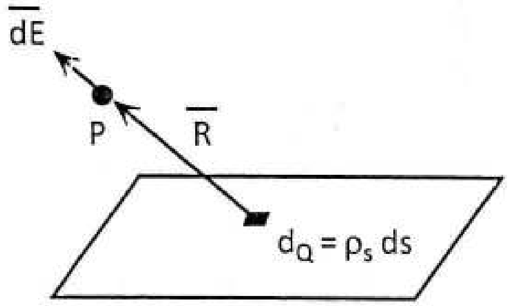
$$Q = \int_S \rho_s \, ds$$

- Each differential charge dQ along the surface results in a differential electric field

$$d\bar{E} = \frac{dQ}{4\pi\epsilon R^2}\,\bar{a}_R = \frac{\rho_s\,ds}{4\pi\epsilon R^2}\,\bar{a}_R$$

- The total field at P is

$$\bar{E} = \int d\bar{E} = \int_S \frac{\rho_s\,ds}{4\pi\epsilon R^2}\,\bar{a}_R$$



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(4) **Volume charge distribution "Surface"**

- The charge is distributed through a specified volume with a volume charge density ρ_v (C/m^3)
- The total charge on the volume

$$Q = \int_V \rho_v \, dv$$

- Each differential charge dQ along the surface results in a differential electric field

$$d\bar{E} = \frac{dQ}{4\pi\epsilon R^2}\,\bar{a}_R = \frac{\rho_v\,dv}{4\pi\epsilon R^2}\,\bar{a}_R$$

- The total field at P is

$$\bar{E} = \int d\bar{E} = \int_V \frac{\rho_v\,dv}{4\pi\epsilon R^2}\,\bar{a}_R$$

3.4 Standard Charge configurations

(1) Infinite Line charge

$$d\bar{E} = \frac{dQ}{4\pi\epsilon R^2}\,\bar{a}_R = \frac{\rho_l\, dl}{4\pi\epsilon R^2}\,\bar{a}_R$$

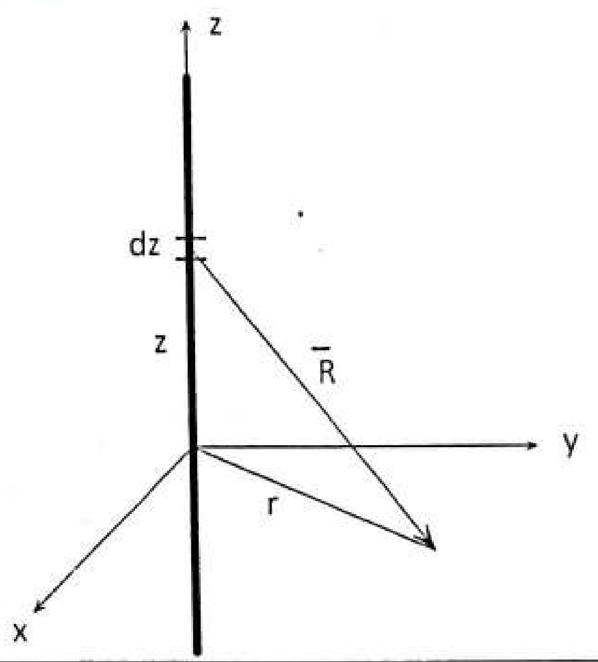
The total field at P is

$$\bar{E} = \int d\bar{E} = \int_L \frac{\rho_l\, dl}{4\pi\epsilon R^2}\,\bar{a}_R = \frac{\rho_l}{4\pi\epsilon}\int_L \frac{dl}{R^2}\,\bar{a}_R$$

$$\bar{R} = z(-\bar{a}_z) + r(\bar{a}_r)$$

$$R = |\bar{R}| = \sqrt{z^2 + r^2}$$

$$\bar{a}_R = \frac{z(-\bar{a}_z) + r(\bar{a}_r)}{\sqrt{z^2 + r^2}}$$



$$\bar{E} = \frac{\rho_l}{4\pi\epsilon}\int_{-\infty}^{\infty} \frac{dz}{(z^2 + r^2)}\left[\frac{r\,\bar{a}_r + z\,(-\bar{a}_z)}{\sqrt{z^2 + r^2}}\right]$$

Due to line symmetry , There is a radial component only

$$\bar{E} = \frac{\rho_l}{4\pi\epsilon}\int_{-\infty}^{\infty} \frac{r\,dz\,\bar{a}_r}{(z^2 + r^2)^{\frac{3}{2}}}$$

$$\bar{E} = \frac{\rho_l\, r}{4\pi\epsilon}\left[\frac{z}{r^2\sqrt{z^2 + r^2}}\right]_{-\infty}^{\infty}\bar{a}_r$$

$$\bar{E} = \frac{\rho_l}{4\pi\epsilon\, r}\,[1 - (-1)]\bar{a}_r$$

$$\boxed{\bar{E} = \frac{\rho_l}{2\pi\epsilon\, r}\,\bar{a}_r}$$

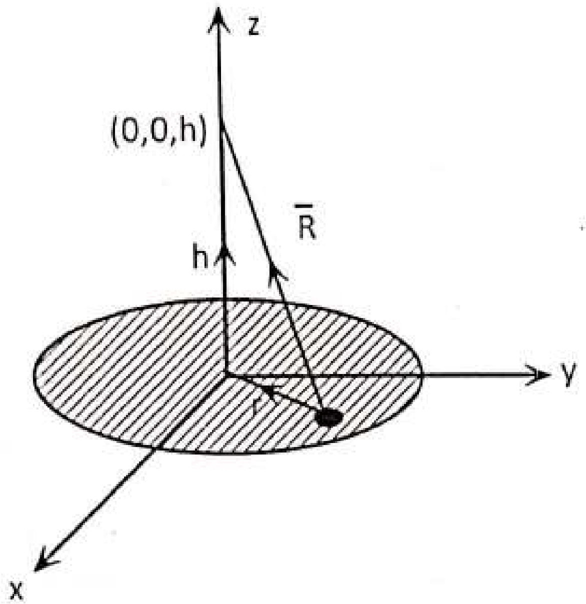
$\bar{E}$: المجال الكهربى عند نقطة بسبب خط طوله ∞ مشحون بـ ρ_l

r : البعد العمودى بين الخط و النقطة

$\bar{a}_r$: متجه وحدة من الخط للنقطة

(2) Infinite plane charge

- Electric field due to circular disk of radius a charged uniformly with surface charge density ρ_s



$$d\bar{E} = \frac{dQ}{4\pi\epsilon R^2}\,\bar{a}_R = \frac{\rho_s\,ds}{4\pi\epsilon R^2}\,\bar{a}_R$$

The total field at P is

$$\bar{E} = \int d\bar{E} = \int_S \frac{\rho_s\,ds}{4\pi\epsilon R^2}\,\bar{a}_R = \frac{\rho_s}{4\pi\epsilon}\int_S \frac{ds}{R^2}\,\bar{a}_R$$

I

$$\boxed{\bar{E} = \frac{\rho_s}{2\epsilon}\left(1 - \frac{h}{\sqrt{a^2 + h^2}}\right)\,\bar{a}_z}$$

- **For infinite sheet**

$$\boxed{\bar{E} = \frac{\rho_s}{2\epsilon}\,\bar{a}_z}$$

Generally

$$\boxed{\bar{E} = \frac{\rho_s}{2\epsilon}\,\bar{a}_n}$$

$\bar{a}_n$: متجه وحدة عمودى من المستوى للنقطة

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7 A plane $y = 3\text{m}$ contains a uniform charge distribution of a density $\rho_s = \left(\frac{10^{-8}}{6\pi}\right) \text{C/m}^2$

Determine $\bar{E}$ at all points

Answer

$$\bar{E} = \frac{\rho_s}{2\epsilon}\,\bar{a}_n$$

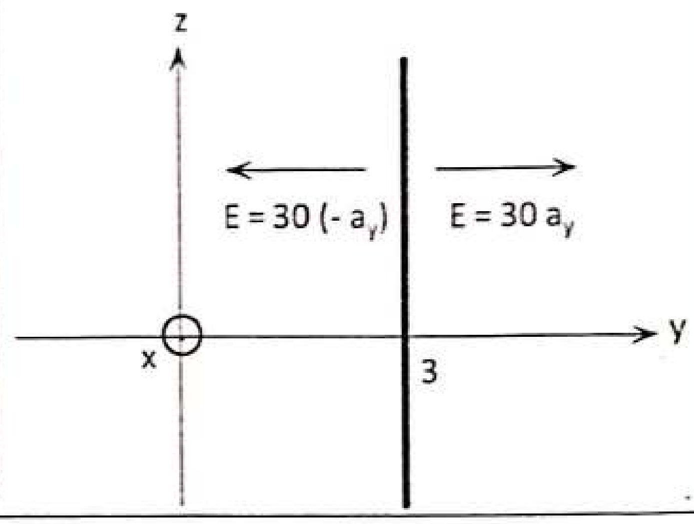
$$\frac{\rho_s}{2\epsilon} = \frac{\frac{10^{-8}}{6\pi}}{8.85 * 10^{-12}} \approx 30$$

for $(y > 3)$

$$\bar{E} = 30\,\bar{a}_y$$

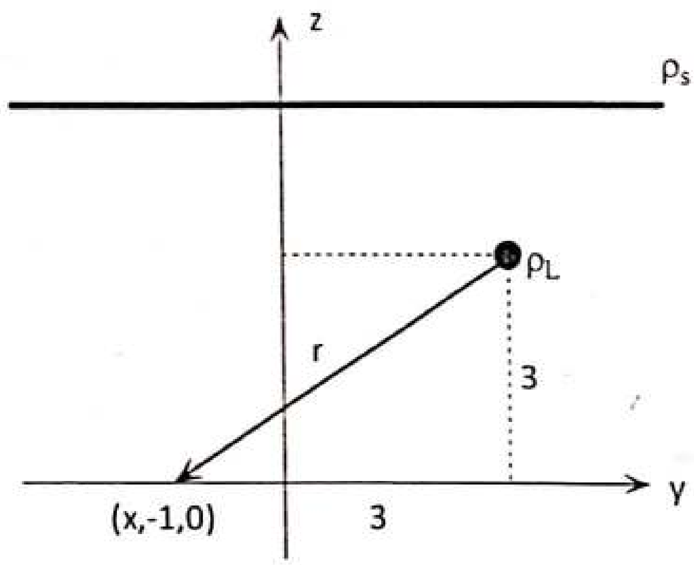
for $(y < 3)$

$$\bar{E} = -30\,\bar{a}_y$$



8 Determine $\bar{E}$ at $(x, -1, 0)$m due to a uniform sheet charge with $\rho_s = \left(\frac{1}{3\pi}\right) nC/m^2$ is located at $Z = 5m$ and a uniform line charge with $\rho_l = \left(\frac{-25}{9}\right) nC/m$ at $z = +3$, $y = 3\ m$.

Answer



$$\bar{E} = \bar{E}_L + \bar{E}_S$$

$$\bar{E} = \frac{\rho_l}{2\pi\epsilon\, r}\,\bar{a}_r + \frac{\rho_S}{2\epsilon}\,\bar{a}_n$$

$$\bar{E} = \frac{-\frac{25}{9} \times 10^{-9}}{2\pi(8.85 \times 10^{-12})\,(5)}\left(\frac{-3\bar{a}_z - 4\bar{a}_y}{5}\right) + \frac{\frac{1}{3\pi} \times 10^{-9}}{2(8.85 \times 10^{-12})}(-\bar{a}_z)$$

$$\bar{E} = -8\bar{a}_y - 12\bar{a}_z$$